TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 2 2020

Section 2 Calculator Assumed (**Scientific only**)

3D Vectors

STUDENT'S NAME _____

DATE: Friday 8 April **TIME:** 50 minutes **MARKS**: 41

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators (**scientific only**), notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Determine the value(s) of λ if the angle between the vectors $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ \lambda \end{pmatrix}$ is $\frac{\pi}{4}$.

2. (12 marks)

Given the two lines $L_1: \quad \underset{\sim}{r} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and $L_2: \quad \underset{\sim}{r} = \begin{pmatrix} -1 \\ -5 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 6 \\ 3 \end{pmatrix}$

(a) Show that the lines intersect at a point P and determine the coordinates of P. [4]

(b) Determine, in the form $ax + by + cz = d$, the equation of the plane containing both the lines L_1 and L_2. [4]

(c) Given that Q_1 and Q_2 are two points on L_1 such that $|PQ_1| = |PQ_2| = 6$, determine the coordinates of Q_1 and Q_2. [4]

3. (4 marks)

Two particles *A* and *B* have position vectors:

$$\underset{\sim}{r}(t) = (20 + 10t)\,\underset{\sim}{i} + 10t\underset{\sim}{j} + (-1.4 + 0.3t)\,\underset{\sim}{k} \quad \text{and}$$

$$\underset{\sim}{r}(t) = (30 + 8t)\,\underset{\sim}{i} + (-90 + 10t)\,\underset{\sim}{j} + (0.9 + 0.2t)\,\underset{\sim}{k}$$

Determine whether the particles collide or their paths cross. State the coordinates of this position.

4. (9 marks)

A plane Π contains two lines:

$$\underset{\sim}{r} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k} + \lambda\left(2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}\right) \text{ and } \underset{\sim}{r} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k} + \mu\left(-\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}\right)$$

(a) Write down a vector equation of the plane Π in the form $\underset{\sim}{r} = \underset{\sim}{a} + \alpha\underset{\sim}{b} + \beta\underset{\sim}{c}$ [1]

(b) The point A with position vector $\begin{pmatrix} 8 \\ 2 \\ c \end{pmatrix}$ lies on the plane. Determine the value of the constant c. [3]

(c) The vector $a\underset{\sim}{i} + b\underset{\sim}{j} + \underset{\sim}{k}$ is perpendicular to the plane. Determine the values of a and b. [3]

(d) State the equation of Π in the form $\underset{\sim}{r} \cdot \underset{\sim}{n} = k$ [2]

5. (11 marks)

The point A has coordinates $(2,0,-1)$ and the plane Π has the equation $\underset{\sim}{r}\cdot\begin{pmatrix}1\\2\\-2\end{pmatrix}=8$. The line through A parallel to the line $\underset{\sim}{r}=\begin{pmatrix}0\\0\\-1\end{pmatrix}+\beta\begin{pmatrix}-2\\1\\2\end{pmatrix}$ meets Π at B and the perpendicular from A to Π meets Π in the point C. [Hint: draw a diagram]

(a) Determine the vector equation of the line through A parallel to the given line, hence determine the coordinates of B. [3]

(b) Determine the vector equation of the perpendicular from A to Π, hence show that the coordinates of C are $\left(\frac{22}{9},\frac{8}{9},\frac{-17}{9}\right)$. [3]

(c) Show that the length of AC is $\frac{4}{3}$. [3]

(d) Determine the value of $\sin(\angle ABC)$. [2]